An iterative method for solving the inverse problem in electrocardiography in normal and fibrillation conditions: A simulation Study

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Abstract. *Electrocardiographic Imaging (ECGI) is a new imaging technique that noninvasively images cardiac electrical activity on the heart surface. In ECGI, a multi-electrode vest is used to record the body-surface potential maps (BSPMs). Then, using geometrical information from CT-scans and a mathematical algorithm we construct the electrical potentials on the heart surface. The reconstruction of cardiac activity from BSPMs is an ill-posed problem. Some algorithms work well in sinus rhythm, but in arrhythmic conditions the reconstruction of the heart potential becomes worst. In this work, we present an iterative mathematical approach based on domain decomposition methods and test it on synthetically data generated using anatomical 43 years old women geometry for normal and fibrillating heart conditions.*

Keywords: Electrocardiography Imaging, inverse problem, domain decomposition, bidomain model, ECG modelling.

1. Introduction

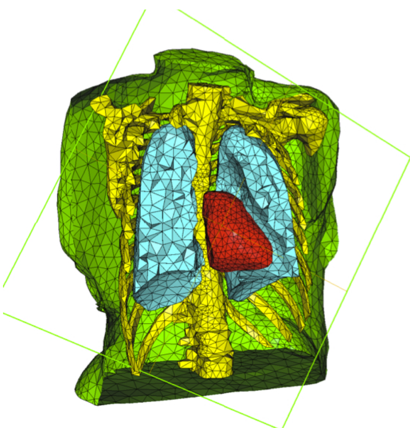
The inverse problem in cardiac electrophysiology also known as electrocardiography imaging (ECGI) is a new and a powerful diagnosis technique. It allows the reconstruction of the electrical potential on the heart surface from electrical potentials measured on the body surface. The computation of the electrical potential on the heart surface giving data on the body surface is known to be ill posed in the sens that a small variation on the –body surface potential could highly modify the solution on the heart surface. In the mathematical community it is known by data completion Cauchy problem. Since 1923, Hadamard [1] have given and exmple illustrating the ill posedness of this problem. In the electrocardiography community, different methods of regularisation have been used and compared in order to solve the problem. These methods include Thikhonov regularisation with L-curve [2], Compiste Residual and smothing Pprator (CRESO) [3], trancated SVD regularisation and other regulatisation techniques see [4] and the refrences in there. In most of the papers in the litterature the used formulation of the inverse probem is based on a transfer matrix that maps the eelctrical potential on the heart onto the body surface. The matrix is computed using the Green formula if the torso is supposed isotropic and homogenous or boundary elements mothod if the torso is supposed to be isotropic and homogeneous by part or in a more genral situation with finite element method in the case where the torso could be supposed anisotropic and/or inhomogenous.

In this work we propose to come back to the original formulation of the problem and propose a new mathematical way to solve the problem. The method that we present in this paper is based on a domain decompostion technique. It has been recently proposed in the international Conference of Domain Decomposition method Zemzemi [5] and tested on cocentric spheres representing the torso. In this work it will be tested on synthetical data generated on a real life human torso.

2. Methods

Anatomical model

The torso geometry was generated from CT images of a 43 years old woman. The DICOM images were segmented using the medical imaging software Osirix. In ordor to take into account the torso heterogeneity we distinguish three different volume regions (lung, bone, and the rest). After generating the surfaces we use INRIA meshing Software MMG3D to generate the 3D volume of the computational mesh. In Figure 1, we show a screenshot of the mesh with the different regions. We assigned isotropic conductivities of 0.389 and 0.2 mS/cm to the lung and bone elements respectively and 2.16 mS/cm in the rest.



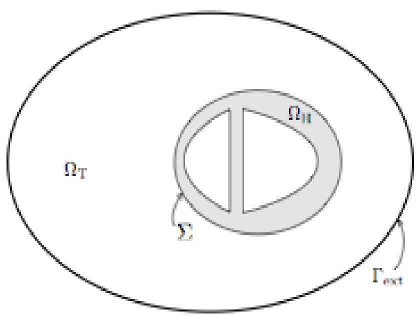


Fig. 1. Left: a cut of the torso computational domain showing the three different regions considered in the torso and the internal and external boundaries of the torso. Right: Two dimensional schematic representation of the heart and torso domains.

Mathematical modeling and numerical method

Supposing that Ω_T denotes the torso domain, Γ_{ext} is the external boundary, Σ is the heart torso interface and u_T is the electrical potential in the torso. The electrical potential on the torso is governed by the diffusion equation. Given the electrical potential ***d*** on the external boundary, the inverse problem in electrocardiography is to find the electrical potential on the heart surface Σ satisfying both Dirichlet and Neumann boundary conditions on Γ_{ext} as shown in Eq. 1

$$\begin{cases} \mathrm{div}(\boldsymbol{\sigma}_{\mathrm{T}} \boldsymbol{\nabla} u_{\mathrm{T}}) = 0, \text{ in } \Omega_{\mathrm{T}}, \\ u_{\mathrm{T}} = d \text{ and } \boldsymbol{\sigma}_{\mathrm{T}} \boldsymbol{\nabla} u_{\mathrm{T}}.\boldsymbol{n} = 0, \text{ on } \Gamma_{\mathrm{ext}}, \\ u_{\mathrm{T}} = ?, \text{ on } \Sigma. \end{cases} \tag{1}$$

Here σ_T denotes the conductivity parameter and depends with the three considered regions. The domain decomposition technique used for solving this inverse problem splits the ill posed problem (1) into two well well posed problems:

$$\begin{cases} \mathrm{div}(\boldsymbol{\sigma}_{\mathrm{T}} \boldsymbol{\nabla} u = 0, \text{ in } \Omega_{\mathrm{T}}, \\ u = d, \text{ on } \Gamma_{\mathrm{ext}}, \\ \boldsymbol{\sigma}_{\mathrm{T}} \boldsymbol{\nabla} u.\boldsymbol{n} = -\boldsymbol{\sigma}_{\mathrm{T}} \boldsymbol{\nabla} v.\boldsymbol{n}, \text{ on } \Sigma. \end{cases} \qquad \begin{cases} \mathrm{div}(\boldsymbol{\sigma}_{\mathrm{T}} \boldsymbol{\nabla} v) = 0, \text{ in } -\Omega_{\mathrm{T}}, \\ \boldsymbol{\sigma}_{\mathrm{T}} \boldsymbol{\nabla} v.\boldsymbol{n} = 0, \text{ on } -\Gamma_{\mathrm{ext}}, \\ v = u, \text{ on } \Sigma. \end{cases} \tag{2}$$

where, $(-\Omega_T)$ and $-\Gamma_{ext}$ are respectively the symmetrical images of Ω_T and Γ_{ext} through the boundary Σ. The well-posed problem (2) could be mathematically seen as the Poincaré-Steklov formulation of a domain decomposition problem. The only non-classic part in this problem is the fact that we have opposite fluxes at the boundary Σ. Different methods have been presented in the monograph [6] in order to solve domain decomposition problems using iterative procedures. In this paper we use the method presented in [5].

3. Results

In order to generate synthetic data we our ECG simulator based on the bidomain model and we generated two cases: In the first case the heart is stimulated in the apex and the electrical wave propagates from apex to base. We call refer to this simulation as a normal case. In the second case we apply a S1-S2 protocol in order to produce a re-entry wave, we refer to this case as a fibrillation case. Further informations about the forward problem modelling could be found in [7]. We extract the BSP and use it as the given data ***d***. Fig. 2 shows snapshots of the electrical potential distribution in the first and second cases for the exact and inverse solution. We remark that the wave front is well captured but in the inverse solution it is much more smoother than in the exact solution.

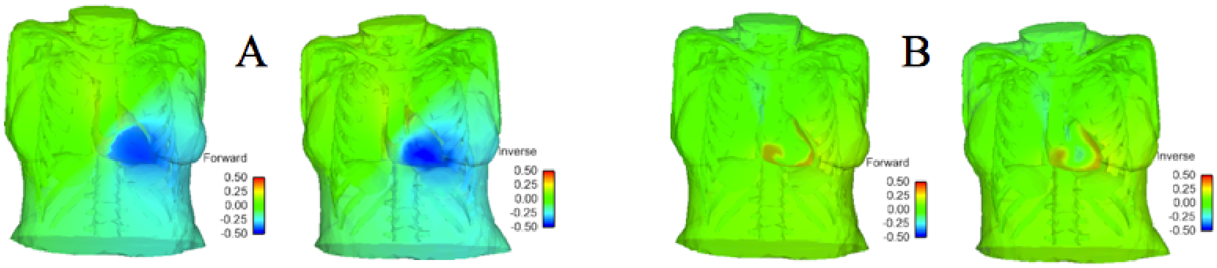


Fig. 2. (Panel A): Snapshots of the potential distribution in the normal case. (Panel B): a snapshot of potential distribution for re-entry case. Forward solution (left) and Inverse solution (right).

In Fig. 3, we show a comparison of the time course of the heart potential between the inverse and the exact solutions at a point located in the right epicardium. The inverse solution is much more accurate in the normal case Fig. 3 (left). In the re-entry case the electrical potential is not accurate in terms of amplitude but looks synchronized with the exact solution. In fact in Fig.4,

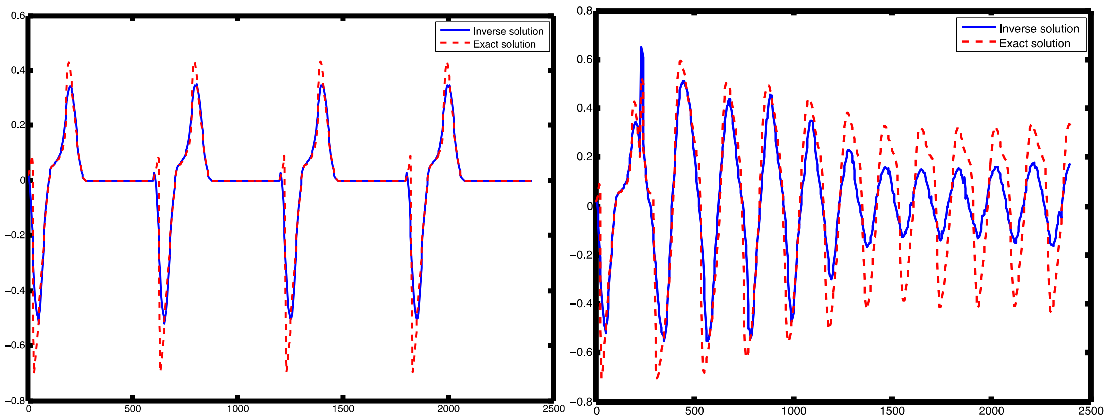


Fig. 3. Comparision of exact (red) and inverse (blue) solutions at a given point in the heart surface for normal (left) and fibrillating (right) heart conditions. X-axis: time (ms) and Y-axis: potential (mV)

we show the evolution of the relative error and its mean in time is 0.45. By contrary, the correlation coefficient looks very Fig. 4 (right), the inverse solution looks at least 93% accurate in terms of the pattern of activation.

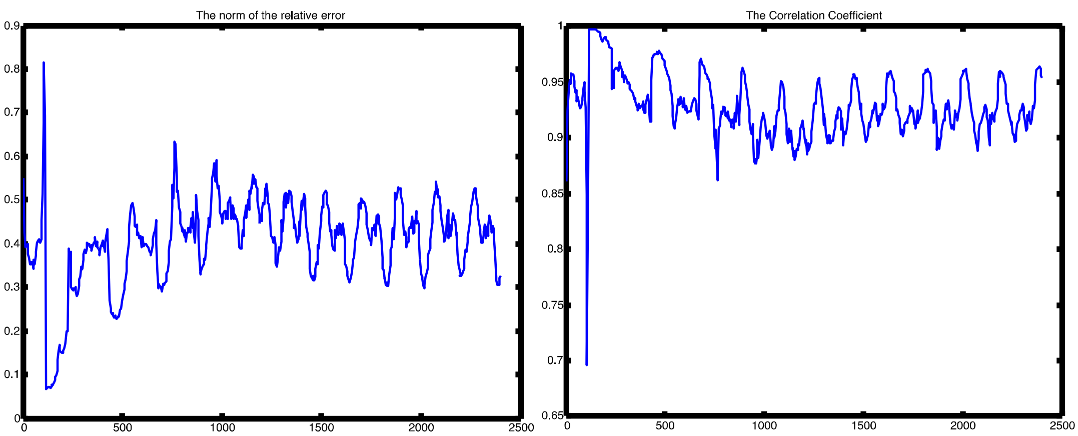


Fig. 4. Time course of the relative error (left) and correlation coefficient (right) computed using the exact and inverse solutions in case of fibrillating heart conditions. X-axis: time (ms) and Y-axis: (dimensionless).

4. Discussion & Conclusions

In this paper, we presented a new approach to solve the inverse problem in electrocardiography. The method is based on a Poincaré-Steklov operator which have been solved using a domain decomposition technique. We used a segmented 3D-anatomical model of a 43 years old women torso. We generated synthetic data using the bidomain model for sinus rhythm and fibrillation conditions. In the normal case the reconstruction is very accurate, we only remarked a slight difference in terms of amplitude. In the re-entry wave case, the accuracy is very low in terms of relative error, mainly because the wave front is highly smoothened and the amplitude of the electrical potential is remarkably lower than the electrical potential of the exact solution. On the contrary, the mean of the correlation coefficient over the time is 0.93. This gives an important potential for this method to be used in clinical applications.

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